An Integrated Total Energy Demand Model
For the Province of Québec

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ABSTRACT

The regulatory environment and government policies applying to energy demand have drawn the attention of model developers. In this paper, we present an integrated total energy demand model and its application to the Province of Québec. Its key determinants are the relative prices of energy sources (coal, electricity, natural gas and oil), the level of economic activity, the number of households and weather. The energy demand, due to dynamic effects, adjusts over time. This econometric model is applied to three economic sectors of the Quebec economy: residential, commercial and industrial. The sample consists of annual time series from 1970 to 1997. The results highlight the roles played by short-run and long-run price and income elasticities. This model is easily used for simulation and forecasting purposes; some examples are featured and they provide some indication of the model's performance as a forecasting tool.

Introduction

It is widely accepted among market analysts that the quantity demanded by consumers for a good or service has an inverse relationship to its price. This general perception derives as much from common sense as from economic theory and basic data observation. Given the significance of this phenomenon, economists have developed a specific concept called price elasticity. The latter measures the relative change (%) in quantity demanded for a good or a service, which results from a relative change (%) in price. One can easily conceive how price elasticities can be useful for studying the expected demand growth of a good or service, and for analyzing the impact of different government actions with respect to prices such as tariffs, taxes or consumption-related subsidies.

The positive link between the consumption of a good or service and the user's income or activity is also widely acknowledged. The significant role played by this link, for analysis or forecasting purposes, comes from its relative size which can also be expressed in terms of income or activity elasticity, i.e. the relative change (%) in quantity demanded which results from a relative change (%) in the user's income or activity.

Empirical assessments of price and income elasticities are not directly available, and must be inferred from observations describing the past behaviour of users within their respective context. Changes to relevant elements in this context, which of course include prices and level of economic activity, induce these changes of behaviour. The main source of information with respect to such behaviour remains the past performances for a given market. Based on this information, we will attempt to obtain the most reliable estimates of price and income elasticities, or any other factor that may seem relevant.

One major difficulty lies in the fact that observations on past behaviour are not the result of controlled experiments, but of real cases where all relevant factors influence the consumer choices simultaneously. This is why economists have developed coherent demand models, which can be estimated with econometric methods in order to obtain estimators. The latter have some desirable statistical properties such as absence of bias, convergence and efficiency. Once the model has been estimated, it may then be used for analysis or forecasting purposes; the latter are the main interests of models' users.

The total energy demand, either with respect to the whole economy or to a specific sector, has garnered widespread attention in the last twenty years as a result of the international oil crises of 1973 and 1979. Today, this topic is still of interest due to global warming, the role played by greenhouse gases and their link to energy consumption. Some studies present a synthesis of previous works, on total energy demand models, namely Ziemba et al. (1980), Bohi and Zimmerman (1984), Donnelly (1987), and Hawdon (1992).

The main purpose of this article is to provide a brief summary of the research I have conducted in collaboration with different authors during the last twenty years on the econometric analysis of total energy demand by sector for the Province of Québec. The discussion below is derived primarily from Arsenault, Bernard, Carr and Genest-Laplante (1995), Bernard and Genest-Laplante (1995) and from some recent works.[1]

This article is divided into three sections: the first section describes the structure of the integrated total energy demand model, the next section presents the empirical results and it focuses on the price and income elasticities as well as on the forecasting properties of this model, and the third section outlines the limitations to the use of such a model. The conclusion contains brief remarks concerning the current use of energy demand models within the emerging regulatory context.

1. Specification of an integrated total energy demand model and of its components

Total energy demand modelling may be applied either to the whole economy or to specific sectors, e.g. residential, commercial and industrial. The sum over all sectors then yields the total demand. It is the latter approach that has been applied here.

Total energy demand with energy source substitution is modelled through two integrated levels: at the first level (aggregate), total energy demand, measured in joules, is made a function of its lagged value, aggregate real energy price, real income and heating degree days. At the second level (disaggregated), market shares held by each energy source (coal, electricity, natural gas and oil) are made functions of the corresponding lagged share and of relative prices of energy sources. Lagged variables at the aggregate and disaggregated levels are introduced to account for dynamic effects over time. Indeed, the use of energy requires complementary equipment, and consumer response to price or income variations may spread out over several periods due to adjustment costs.

More formally, the integrated total energy demand model by sector can be written in the following terms:

$$MS\phi_t = f(MS\phi_{t-1}, PC_t, PEL_t, PNG_t, PO_t) \quad (1)$$

$$PEN_t = \sum_{\phi} MS\phi_t \text{ x } P\phi_t \quad (2)$$

$$EN_t = h\,(EN_{t-1}, PEN_t \,/\, PI_t, Y_t, DD_t) \tag{3}$$

$$Q\phi_t = MS\phi_t \text{ x } EN_t \tag{4}$$

where $\phi = \mathrm{Coal}\,(C)^2$, Electricity (EL), Natural Gas (NG), Oil (O);

$MS\phi_t =$ market share (%) of energy source ϕ in year t;

$PEN_t =$ price ($/joule) of total energy in year t;

$P\phi_t =$ price ($/joule) of energy source ϕ in year t;

$EN_t =$ total energy consumption (joules) in year t;

$PI_t =$ general price index in year t;

$Y_t =$ real income in year t;

$DD_t =$ heating degree days in year t;

$Q\phi_t =$ consumption (joules) of energy source ϕ in year t.

Equations (1) to (4) form an integrated two-level model of total energy demand and of its decomposition into separate energy sources. The set of share equations held by each energy source incorporates the substitution possibilities among energy sources based on their relative prices. These share equations are used to obtain the aggregate energy price (2), which is simply the weighted sum of the prices of different sources. This aggregate energy price determines the level of total demand (3) together with other variables such as real income and degree days[3]. Share equations (1) and total energy demand (3) are combined to obtain the demand of each energy source (4).

This two-level integrated model provides a tool which can be easily used for policy simulation or for forecasting. The substitution effects among energy sources (set of equations (1)) and between total energy and the other goods (equation (3)) are incorporated explicitly. Furthermore, real income also has an impact on energy consumption.[4] The exogenous variables that determine energy consumption are the relative prices of energy sources and real income. At each period, lagged variables are also known variables which determine the current demand level.

At the estimation stage, the set of energy market share functions (1) receives a semi-logarithmic form in terms of the relative prices of energy sources. Since the set of market shares is a partition among energy sources, some restrictions must be imposed to ensure that the sum of shares adds up to one:

i. each market share equation is homogenous of degree zero in the prices of energy sources;
ii. the coefficient of the lagged share variable is the same for each equation;
iii. the effect of the price of source i on market share held by source j is the same as the effect of the price of energy source j on share of source i;
iv. the intercepts and the coefficient of the lagged share variables add up to one.

At the estimation stage, the function (3) takes a logarithmic form. This implies that price and income elasticities of total energy demand may be calculated directly. Since the effects are spread over time, one has to make a distinction between the short-run elasticity, capturing the effect obtained during the current year, and the long-run elasticity, representing the cumulative effect once the complete adjustment has been captured over several periods. More formally, the elasticity of total energy demand with respect to its price is:

$$\varepsilon_{EN:PEN} = \frac{PEN}{EN} \text{ x } \frac{dEN}{dPEN}$$

(5)

The elasticity with respect to income can be expressed in the following terms:[5]

$$\varepsilon_{EN:Y} = \frac{Y}{EN} \text{ x } \frac{dEN}{dY}$$

(6)

It should be pointed out that total energy and its components are measured in joules, i.e. in terms of thermal equivalence. Market shares (%) are therefore expressed in terms of thermal equivalence rather than expenditure shares. Since the warnings of Turvey and Nobay (1969), economists have recognized that for theoretical reasons, it is more appropriate to use expenditure shares.[6] The use of thermal weights can introduce systematic biases which are transmitted to price and income elasticities estimates.[7] However, there are practical reasons for measuring energy on the basis of thermal equivalence; indeed, in their analyses and forecasts, federal and provincial governments as well as regulatory agencies base their measure of energy consumption on thermal equivalence. Using this approach can therefore make the comparisons with the other models and their results easier.

2. Estimation and simulation

The above model is estimated using annual time series for the Province of Québec, which run from 1970 to 1997. Data have been gathered for separate energy sources (coal, electricity, natural gas and oil products) in three sectors (residential, commercial and industrial). For the most part, the statistical data are taken from publications released by Statistics Canada.

2.1 Estimation results

The ordinary least squares (OLS) estimation method is applied to equation (3) representing total energy consumption by sector, and the results appear in Table 1. With few exceptions, the results are satisfactory when they are assessed in terms of some commonly used statistical criteria. All price and income coefficients display signs that are expected on *a priori* grounds. The lagged dependent variables have a high level of significance and they take values between zero and one; these values indicate the presence of stable dynamic adjustments. R^2 coefficients take high values and Durbin-h statistics are low; the only exception appears in the industrial sector where the error terms have autocorrelation.

Zellner's seemingly unrelated regression (SUR) procedure is applied to the set of market share equations (1), for which the results are shown in Table 2. One can see that the coefficients of the lagged dependent variables are all very high, thus indicating a very slow adjustment process of market shares. The coefficients of relative prices variables for energy sources are generally significant and indicate the presence of substitution among energy sources.

2.2 Total energy demand price and income elasticities

From the estimation results shown in Table 1, we can calculate the short-run and the long-run total energy demand price and income elasticities directly by sector. Table 3 displays the estimates of price elasticities, which are all less than one in absolute value for the short run and the long run; this fact is particularly significant in the industrial sector, thus indicating that energy consumption responds weakly to price changes.[8] Income elasticities are relatively high in the commercial and industrial sectors and are found to be close to one over the long run. This implies that for both these sectors, energy consumption follows the level of economic activity in the long run.

2.3 Forecasting and simulation

The integrated total energy demand model presented above may easily be used for simulation or forecasting purposes. We only need to insert into the model the exogenous variables, which, in this case, are the prices of the energy sources, the level of economic activity and the number of households. These variables are considered to be the basic determinants of the growth of total energy demand.

To illustrate the use of this model, two simulations have been performed over the sample period, which runs from 1970 to 1997. In the first simulation, the exogenous variables observed as well as the lagged variables calculated from the previous year determine the expected total energy demand for the current year. Since we are using the observed exogenous variables, the forecasting errors derive from the model itself rather than from the explanatory variables. Furthermore, by using the calculated lagged variables, we can analyze the model's propensity to reproduce more or else rapidly the real energy demand. In the second simulation, the only explanatory variables being used are the observations, which also include the lagged dependent variables. In this case, the emphasis is on the short-run forecasting performance.

To analyze the model's forecasting properties, the Theil coefficient (1966) and its three-part decomposition is used. The Theil coefficient is derived from the sum of relative squared forecasting error:

$$U^2 = \sum_{t=1}^{T} \left(P_t - A_t\right)^2 / \sum_{t=1}^{T} A_t^2 \tag{7}$$

where P_t = predicted value in period t;

A_t = actual value in period t

U may therefore be interpreted as the average relative error (%) per year. Theil (1966) has also shown that U^2 can be decomposed in three parts expressed in relative terms:

$$U^m + U^s + U^r = 1$$

where U^m = the part of the forecasting error due to the difference between the mean of P_t and the mean of A_t;

U^s = the part of the forecasting error due to the model structure;

U^r = the part of the forecasting error due to residual error.

For forecasting purposes, in an ideal forecasting model, U^2 would be the smallest possible, i.e. the relative forecasting error would be the lowest possible. For a given U^2 , U^m and U^s should be close to zero and U^r close to one.

Table 4 presents the estimates of these coefficients for the two simulations discussed above. One can see that the average relative forecasting error is approximately 2% in the residential sector, 4% for the commercial sector, and 6 to 11.5% in the industrial sector. Except for simulation 1 in the industrial sector, the main source of error is the residual error, indicating that U^r is close to one.

3. A few limitations

The forecasting properties of an econometric model depends both on the quality of the exogenous variables and on the stability of the model structure over the long run. The data with respect to the expected evolution of the exogenous variables, i.e. the prices of the energy sources and the economic conditions, generally come from the opinions of experts. Even experts make mistakes. The structural stability of the model may be submitted to statistical analysis. However, with the use of annual series, as it is the case here, the data are gathered slowly and several years have to go by before conclusive tests may be conducted with respect to this issue.

Conclusion

In the aftermath of the international oil crises in 1973 and 1979, governments regulated oil and natural gas prices. Since 1985, these prices have been deregulated in Canada. Today, electricity production is being open to market forces. Consequently, governments are reducing their regulatory presence in the energy sector, and the interest in forecasting energy demand growth for this purpose has diminished. On the other hand, new issues are emerging such as global warming. Energy demand growth, especially in the form of fossil fuels, is directly impacted. For this reason, developing appropriate tools for analyzing and forecasting energy demand is still important today. The above model may serve this purpose.

TABLE 1

Total energy demand

EXPLANATORY VARIABLES	RESIDENTIAL	COMMERCIAL	INDUSTRIAL
Intercept	2.731 (3.81)[a]	1.794 (1.16)	1.909 (1.68)
Lagged dependent	0.643 (8.79)	0.377 (2.60)	0.366 (2.08)
Real price of energy	-0.259 (-4.90)	-0.328 (-3.53)	-0.066 (1.70)
Real disposable income per household	0.125 (1.18)	-	-
Commercial GDP	-	0.577 (3.76)	-
Industrial GDP	-	-	0.614 (3.30)
Heating degree days	0.409 (4.96)	0.660 (3.13)	-
R^2 **Durbin-h** **Number of observations**	0.999 -0.77 28	0.998 -0.25 28	0.832 13.96 28

a) The t-statistics appear in parentheses.

TABLE 2
Market share equations

Explanatory variables	Market Shares		
	ELECTRICITY	**OIL**	**COAL**
I. Residential sector			
Intercept	0.089 (6.93)[a]	-0.016 (-2.16)	-
Dependent	0.929 (60.05)	0.929 (60.05)	-
Electricity price[b]	-0.053 (-5.06)	0.041 (4.0)	-
Oil price[b]	0.04 (4.01)	-0.06 (-4.93)	-
II. Commercial sector			
Intercept	0.103 (4.36)	-0.012 (-0.89)	-
Dependent	0.886 (36.25)	0.886 (36.25)	-
Electricity price[b]	-0.041 (-2.49)	0.035 (2.43)	-
Oil price[b]	0.035 (2.43)	-0.100 (-4.79)	-
III. Industrial sector			
Intercept	0.038 (2.64)	-0.017 (-1.15)	0.008 (3.16)
Dependent	0.941 (32.91)	0.941 (32.91)	0.941 (32.92)
Electricity price[b]	-0.010 (-0.78)	0.038 (2.54)	-0.014 (-4.15)
Oil price[b]	-0.038 (2.54)	-0.081 (-3.37)	0.012 (2.02)
Coal price[b]	-0.014 (4.15)	0.012 (2.02)	-0.003 (0.77)

a) The t-statistics appear in parentheses.

b) The price of the indicated energy source is relative to the price of natural gas.

TABLE 3

Total energy demand price and income elasticities

	PRICE		INCOME	
	SR	**LR**	**SR**	**LR**
Residential	-0.25	-0.73	0.13	0.35
Commercial	-0.33	-0.53	0.58	0.93
Industrial	-0.07	-0.10	0.01	0.97

SR: Short run

LR: Long run

TABLE 4

Forecasting: Theil's inequality coefficient and its decomposition

	Residential Simulation		Commercial Simulation		Industrial Simulation	
	1	**2**	**1**	**2**	**1**	**2**
U	0.013	0.021	0.038	0.039	0.115	0.065
$\mathbf{U^m}$	0.084	0.001	0.070	0.000	0.038	0.001
$\mathbf{U^s}$	0.002	0.001	0.000	0.010	0.264	0.058
$\mathbf{U^r}$	0.914	0.998	0.930	0.990	0.698	0.942

NOTES

1. I wish to express my thanks to Eric Boudreault, Valérie Caverivière and Pierre-Renaud Tremblay for assisting me through this research.

2. Coal appears only in the industrial sector.

3. Degree days do not appear as an explanatory variable in the industrial sector.

4. In the residential sector, total energy demand and real disposable income are expressed on a per household basis, so that the number of households is taken into account.

5. Because of the two-level structure where income doesn't appear in share equations, energy source elasticities with respect to income are the same as the income elasticity appearing in total energy demand (3). The price elasticities of the energy sources are, on the other hand, more complex to calculate.

6. For applications of energy expenditure share models to the Province of Québec, see Bernard, Lessard and Thivierge (1986) for the commercial sector, and Bernard, Lemieux and Thivierge (1987) for the residential sector.

7. See Bernard and Cauchon (1987) for further analysis of the empirical size of theses bias for the Province of Québec.

8. It is also possible to calculate the price elasticities for each energy source. See Bernard and Genest-Laplante (1995).

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